

Semirings which are distributive lattices of weakly left k -Archimedean semirings

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Abstract. We introduce a binary relation \xrightarrow{l} on a semiring S , and generalize the notion of left k -Archimedean semirings and introduce weakly left k -Archimedean semirings, via the relation \xrightarrow{l} . We also characterize the semirings which are distributive lattices of weakly left k -Archimedean semirings.

1. Introduction

The notion of the semirings was introduced by Vandiver [12] in 1934. The underlying algebra in idempotent analysis [6] is a semiring. Recently idempotent analysis have been used in theoretical physics, optimization etc., various applications in theoretical computer science and algorithm theory [5, 7]. Though the idempotent semirings have been studied by many authors like Monico [8], Sen and Bhuniya [11] and others as a $(2, 2)$ algebraic structure, idempotent semirings are far different from the semirings whose multiplicative reduct is just a semigroup and additive reduct is a semilattice. So for better understanding about the abstract features of the particular semirings \mathbb{R}_{max} (Maslov's dequantization semiring), Max-Plus algebra, syntactic semirings we need a separate attention to the semirings whose additive reduct is a semilattice. From the algebraic point of view while studying the structure of semigroups, semilattice decomposition of semigroups, an elegant technique, was first defined and studied by Clifford [4]. This motivated Bhuniya and Mondal to study on the structure of semirings whose additive reduct is a semilattice [1, 2, 9, 10]. In [1], Bhuniya and Mondal studied the structure of semirings with a semilattice additive reduct. There, the description of the least distributive lattice congruence on such semirings was given. In [10], Mondal and Bhuniya gave the distributive lattice decompositions of the semirings into left k -Archimedean semirings. In this paper we generalize the notion of left k -Archimedean semirings introducing weakly left k -Archimedean semirings, analogous to the notion of weakly left k -Archimedean semigroups [3] and characterize the semirings which

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